\[ V^\pi(s) = \sum_{t=1}^{\infty} E \left\{ \gamma^{t-1} r_t \mid s_0 = s \right\} \] (1)
\[ = \sum_a \pi(s, a) \left[ R(s, a) + \gamma \sum_{s'} P(s, s', a)V^\pi(s') \right] \] (2)

\[ d^\pi(s') = \lim_{t \to \infty} Pr \{ s_t = s' \mid s_0, \pi \} \quad \text{(does not depend on } s_0) \] (3)
\[ = \sum_s d^\pi(s) \sum_a \pi(s, a)P(s, s', a) \] (4)

\[ \rho^\pi = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} r_t \quad \text{(does not depend on } s_0) \] (5)
\[ = \sum_s d^\pi(s) \sum_a \pi(s, a)R(s, a) \] (6)

In trying to form an overall discounted performance measure for \( \pi \), can we use \( J(\pi) = \sum_s d^\pi(s)V^\pi(s) \)? It turns out we then end up with no effect of the discounting:

\[ J(\pi) = \sum_s d^\pi(s)V^\pi(s) \] (7)
\[ = \sum_s d^\pi(s) \sum_a \pi(s, a) \left[ R(s, a) + \gamma \sum_{s'} P(s, s', a)V^\pi(s') \right] \] (8)
\[ = \rho^\pi + \gamma \sum_s d^\pi(s) \sum_a \pi(s, a) \sum_{s'} P(s, s', a)V^\pi(s') \] (9)
\[ = \rho^\pi + \gamma \sum_{s'} V^\pi(s') \sum_s d^\pi(s) \sum_a \pi(s, a)P(s, s', a) \] (10)
\[ = \rho^\pi + \gamma \sum_{s'} V^\pi(s')d^\pi(s') \] (11)
\[ = \rho^\pi + \gamma J(\pi) \] (12)
\[ = \rho^\pi + \gamma \rho^\pi + \gamma^2 J(\pi) \] (13)
\[ = \rho^\pi + \gamma \rho^\pi + \gamma^2 \rho^\pi + \gamma^3 \rho^\pi + \cdots \] (14)
\[ = \frac{1}{1 - \gamma \rho^\pi} \] (15)

which is basically a scaled \( \rho^\pi \), with no effect of discounting.