personal perspective

- there is a *science of mind* that is neither natural science nor applications technology
- e.g., Marr’s “computational theory” level
- a mind is something more usefully thought of in terms of goals than mechanisms
- goals can be well thought of as rewards
- reinforcement learning is the beginning of a science of mind
- intrinsic motivation is part of a science of mind
Core Learning Algorithms for Intrinsically Motivated Agents

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with thanks to
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summary

- a technical breakthrough leading to a new family of reinforcement learning algorithms
- temporal-difference (TD) learning with approximations is now straightforward
- it is now practical to learn in parallel about many predictions at once
- parallel prediction learning may be key to understanding mind, brain, and intrinsic motivation
knowledge is *predictions*

• of what will happen
• of what you could cause to happen
  • at various time scales
  • conditional on actions or courses of action
predictions can be more powerful than you think

- not just a “saying before” of what the sensory signals will be
- all scientific knowledge can be expressed as predictions
- predictions can be about the outcomes of extended courses of behavior (options)
- all the little things you know can be well thought of as prediction
much of mind is about prediction

- *perception* and *state representation* can be thought of as making predictions
- *models the world* and *cause and effect* can be thought of in terms of predictions
- *planning* can be thought of as composing predictions to anticipate possible futures, and then choosing among them
- *learning value functions* (and thus much of conventional RL) is learning predictions
• predictions are the coin of the mental realm

• thus, we should be focused on machinery for efficiently learning predictions
parallel prediction demons

• we should be able to make *and learn* lots of predictions at once, in parallel

• as in *parallel prediction demons*

• every demon should be able to learn on every step

• this has always been the promise of off-policy temporal-difference learning
but this promise has been unfulfilled

- there has been no practical algorithm for parallel prediction learning

- previous methods were too complex (LSTD, iLSTD), restricted to table lookup (Q-learning), not parallel (Monte Carlo, Sarsa), too slow (importance sampling), or had weak approximators (averaging)
until now

• now, for the first time, it is practical and straightforward to do massive, in parallel, prediction learning

• with new gradient-based TD algorithms
  • GTD, TDC (NIPS-08, ICML-09, NIPS-09)
  • GQ (submitted)
  • Actor-critic-option algorithms (in prep)
Perhaps key to mind is being able to make and learn a lot of predictions in parallel, which is what TD learning was made for, and which we can now finally do, with GQ.

- Setting (online prediction learning)
  - Watkins’s linear $Q(\lambda)$
- GQ inputs and outputs
- The GQ algorithm
- Example uses of GQ; including for IM
Setting
real-time, incremental

- all predictions are *made* and *learned* on every time step
- 100 times a second
- constant-time computation per step
- constant memory
a continual stream of experiential data

• every time step (say 10ms) we receive a new sensory observation $o_t$ and take an action $a_t$

• we also update an agent state representation, $s_t$:

$$s_{t+1} = u(s_t, a_t, o_{t+1})$$

• $s_t$ is whatever the agent uses to pick $a_t$, i.e., there exist probabilities

$$\Pr[a_t = a \mid s_t = s]$$

• we place no constraints on $s$, $u$, or $b$
approximation architecture is fixed and linear (for now)

- state-action pairs map in a fixed way to feature vectors
  \[ s_t a_t \rightarrow \mathbf{x}_t = (x_t(1), x_t(2), \ldots, x_t(n)) \in \mathbb{R}^n \]

- which map linearly to scalar predictions
  \[ p_t^i = \mathbf{w}_t^i \cdot \mathbf{x}_t = \sum_j w_t^i(j) x_t(j) \quad \mathbf{w}_t^i \in \mathbb{R}^n \]

- where the weights, \( \mathbf{w}_t^i \), are what is learned

- we consider one prediction, and drop the “\( i \)"
Watkins’s $Q(\lambda)$
Watkins’s $Q(\lambda)$ semantics (what is learned)

Learns an approximation to the optimal action-value function:

$$p_t = w_t \cdot x_t \approx Q^*(s_t, a_t) \in \mathcal{R}$$

where

$$Q^*(s, a) = \max_{\pi} E_{\pi} \left[ r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \cdots \mid s_t = s, a_t = a \right]$$

- **Policy for picking $a_k$, $k \geq t$**
- **Discount rate** (e.g., $\gamma = 0.9$)
Watkins’s $Q(\lambda)$ learning rule

$$\delta_t = r_{t+1} + \max_a \gamma w_t \cdot x_{t+1} - w_t \cdot x_t$$  
TD error, $\delta_t \in \mathbb{R}$

$$e_t = \begin{cases} 
\gamma \lambda e_t + x_t & \text{if } a_t = \arg\max_a w_t \cdot x(s_t, a) \\
 x_t & \text{otherwise}
\end{cases}$$  
eligibility trace, $e_t \in \mathbb{R}^n$

$$w_{t+1} = w_t + \alpha \delta_t e_t$$  
trace decay rate, $\lambda \in [0,1]$

• unsound (may diverge)
• traces frequently cut
• off-policy - learns the optimal policy and values
• independent of the behavior policy
Watkins’s $Q(\lambda)$ provides a hint of the massive generalizations that are possible

- you could learn simultaneously about multiple, different
  - reward functions
  - discount rates
  - non-greedy target policies
  - state-dependent terminations
- you could predicted outcomes as well as rewards
**GQ(λ,π,β,z,r) semantics**

- **target policy**, \( \pi : A \times S \rightarrow [0, 1] \), \( \sum_a \pi(a | s) = 1 \)
- **termination probability**, \( \beta : S \rightarrow [0, 1] \)
- **initiation set (set of interest)**, \( I : S \times A \rightarrow [0, 1] \)
- **transient target function**, \( r : S \rightarrow \mathbb{R} \)
- **outcome target function**, \( z : S \rightarrow \mathbb{R} \)
- **eligibility-trace decay function**, \( \lambda : S \rightarrow [0, 1] \)

\[
p_t \approx E_{\pi,\beta,\lambda} \left[ r_{t+1} + r_{t+2} + \cdots + r_T + \begin{array}{c} \text{or} \\ \text{or} \end{array} \begin{array}{c} z_T \\ w \cdot x_T \end{array} \right]_{s_t, a_t, T = \text{time of termination by } \beta \text{ or truncation by } \lambda}
\]
notational shortcuts

\[ \beta_t = \beta(s_t) \]

\[ I_t = I(s_t) \]

\[ r_t = r(s_t) \]

\[ z_t = z(s_t) \]

\[ \lambda_t = \lambda(s_t) \]

\[ \overline{\mathbf{x}}_t = \sum_a \pi(a \mid s_t) \mathbf{x}(s_t, a) = E \left[ \mathbf{x}_t \mid a_t \sim \pi \right] \]
### GQ learning rule

\[
\delta_t = r_{t+1} + \beta_{t+1} z_{t+1} + (1 - \beta_{t+1}) w_t \cdot \overline{x}_{t+1} - w_t \cdot x_t
\]

\[
e_t = (1 - \beta_t) \lambda_t \frac{\pi(a_t | s_t)}{b(a_t | s_t)} e_{t-1} + I x_t
\]

\[
\Delta w_t = \alpha \left[ \delta_t e_t - (1 - \beta_{t+1})(1 - \lambda_{t+1})(v_t \cdot e_t) \overline{x}_{t+1} \right]
\]

\[
\Delta v_t = \alpha \eta \left[ \delta_t e_t - (v_t \cdot x_t) x_t \right]
\]

- everything is \( O(\#\text{features}) \)
- everything is well-defined and readily available
- similarities to expected Sarsa, with \((1 - \beta)\) in place of \( \gamma \)
void GQlearn(x, x_bar, I, \rho, \beta, z, r, \lambda) { /* all except xs are scalar */
    static double w[n], v[n], e[n];
    double \alpha = 0.0001, \eta = 1.0, dotux, dotue;
    \delta = r + \beta * z + (1-\beta) * \text{dot}(w, x_{\text{bar}}) - \text{dot}(w, x);
    for (i=0; i<n; i++) e[i] = \rho * e[i] + I * x[i];
    dotve = \text{dot}(v, e);
    dotvx = \text{dot}(v, x);
    for (i=0; i<n; i++) {
        w[i] += \alpha * (\delta * e[i] - (1-\beta) * (1-\lambda) * \text{dotve} * x_{\text{bar}}[i]);
        v[i] += \alpha * \eta * (\delta * e[i] - \text{dotvx} * x[i]);
        e[i] *= (1-\beta) * \lambda;
    }
}
There exists a scalar Bellman-error objective function

\[ J(w) = \left\| p_w - \Pi T^{\lambda \beta}_\pi p_w \right\|^2_{D_b \cdot I} \]

such that

\[ E_b \left[ \Delta w \right] \propto - \nabla_w J(w) \]

which guarantees convergence to \( J(w) = 0 \)
(under step-size conditions)
Example uses of GQ
what kind of things might we do with these demons?

- make everything a reward (for some demon)
- learn an option to achieve it
- learn a detector for the ability to achieve it
- take hand-coded options, learn about their outcomes
- learn models of options suitable for planning
- guide behavior by the demons’ learning progress (“learning feels good”)
bump anticipators

Continuously predict imminent bumps at various time scales under the behavior policy

\[ \pi(a \mid s) = b(a \mid s) \]

\[ \beta(s) = 0.1 \quad \text{(bump in next 10 steps = 0.1 seconds)} \]
\[ \beta(s) = 0.02 \quad \text{(bump in next 50 steps = 0.5 seconds)} \]
\[ \beta(s) = 0.002 \quad \text{(bump in next 500 steps = 5 seconds)} \]

\[ I(s) = 1.0 \]
\[ r(s) = 0 \]
\[ z(s) = \| \text{accelerometer}(s) \| \]
\[ \lambda(s) = 0.9 \]

These 3 predictions could then be added to the agent state, used to make decisions
“near something” detector

Can I get any IR sensor to give a sustained high reading without moving very much?

Add detector \( d(s) \) for high IR reading for 0.5 seconds

\[
\pi(a \mid s) = 1 \quad \text{if} \quad a = \arg \max \limits_{a'} \mathbf{w} \cdot \mathbf{x}(s, a'), \quad \text{otherwise 0}
\]

\[
\beta(s) = d(s) (+ 0.01 \text{ if wheels are moving})
\]

\[
I(s) = 1.0
\]

\[
r(s) = 0
\]

\[
z(s) = d(s)
\]

\[
\lambda(s) = 0.95
\]

Does not need to be run to completion
other possibilities

• is there a ball (concave object) present?

• am I stuck? (what would happen to wheel motion when torque is applied after/for several time steps?)

• can I get the rattle to sound? How?

• target policies with constant actions may be useful
learning a model of an option

For planning we need *models* of possible courses of action (e.g., wall following)

Each such *option model* is a bunch of predictions for the option’s $\pi, \beta, I$:

Predictions whose outcome targets are the elements of the agent state: $r^i(s) = 0, z^i(s) = s(i)$

One more prediction whose transient target is $r(s) - \bar{r}$, the deviation of the current *real reward* from the long-term average real reward

This form is necessary and sufficient for planning
intrinsic motivation

• imagine one million prediction demons, all learning in parallel

• for various random or cleverly chosen options and target functions

• imagine each can measure its learning progress

• use the sum-total learning progress as intrinsic reward to direct the behavior policy

• weed and refine the set of demons, then repeat
conclusions

• *prediction demons* are a powerful language for learning and representing knowledge

• *prediction demons* can learn online, in parallel, and computationally efficiently

• they can certainly be used to learn a lot of stuff about one’s world

• probably more than any previous AI

• and they can be a powerful, lightweight substrate for intrinsic motivation systems
• thank you for your attention