CMPUT 609 Written 4: Temporal-Difference Learning
Due: Friday October 9 in class (no slip days)

There are a total of 66 points available on this assignment.

1. **[8 points] (greedy wrt a value function)**

   **True or False and justify your answer** (8 points total):

   (a) **T**  **F** (4 pts) If a policy is greedy with respect to the value function for the equiprobable random policy, then it is an optimal policy.

   (b) **T**  **F** (4 pts) If a policy \( \pi \) is greedy with respect to its own value function, \( v_\pi \), then it is an optimal policy.
2. [22 points] (all about $v_\pi$)

Consider the value function $v_\pi$ for a stochastic policy $\pi$ and a continuing finite Markov decision process with discounting.

(a) [3 pts] Give an equation defining $v^\pi(s)$ in terms of the subsequent rewards $R_{t+1}, R_{t+2}, \ldots$ that would follow if the MDP were in state $s$ at time $t$. (If you choose to write it in terms of the return, $G_t$, define your return notation in terms of the underlying rewards.)

(b) [3 pts] Sketch the backup diagram for the dynamic programming algorithm (full backup) for $v_\pi$. Find a place to attach the labels $s$, $a$, $r$, and $s'$. 

(c) [4 pts] What is the Bellman equation for $v_\pi$? Write it in an explicit form in terms of $p(s', r|s, a)$ so that no expected value notation appears.
(d) [4 pts] Consider the simplest dynamic-programming algorithm for computing $v_\pi$. An array $V(s)$ is initialized to zero. Then there are repeated sweeps through the state space, with one update to an array element done for each state. What is that DP update?

(e) [4 pts] Consider the simplest temporal-difference learning method for estimating $v_\pi$ from experience. An array $V(s)$ is initialized to zero. Then an infinite sequence of experience, $S_0, A_0, R_1, S_1, A_1, R_2, S_2, \ldots$, is processed, with one update to $V(S_t)$ done for each transition. What is the equation for that TD update?

(f) [4 pts] Now consider the simplest Monte Carlo learning method for estimating $v_\pi$ from episodic experience. An array $V(s)$ is initialized to zero. Then an infinite sequence of episodes is experienced, where an individual episode of experience is denoted $S_0, A_0, R_1, S_1, A_1, R_2, S_2, \ldots, R_T, S_T$, where $T$ is the final time step of the episode, $S_T$ is the terminal state, and the value of all terminal states is taken to be zero. When an episode is processed, one update to $V(S_t)$ is made for each time step $t < T$. What is the equation for that Monte-Carlo update? (If you choose to write it in terms of the return, define your return notation in terms of the underlying rewards.)
3. [24 points] (episodic example of TD and MC)

Suppose you observe the following 9 episodes generated by an unknown Markov reward process, where A and B are states and the numbers are rewards:

<table>
<thead>
<tr>
<th>Episode</th>
<th>Reward</th>
</tr>
</thead>
<tbody>
<tr>
<td>A,0,B,4</td>
<td>B,0,A,1,B,2</td>
</tr>
<tr>
<td>A,2</td>
<td>B,0,A,2</td>
</tr>
<tr>
<td>A,1,B,0,A,2</td>
<td>B,2</td>
</tr>
</tbody>
</table>

(a) (8 pts) Give the values for states A and B that would be obtained by the batch first-visit Monte-Carlo method using this data set (assuming no discounting). You may express your answer using fractions. Briefly explain how you arrived at your answer.

(b) (8 pts) If you were to form a maximum-likelihood model of a Markov reward process on the basis of these episodes (and these episodes alone), what would it be? (sketch its state-transition diagram with transition probabilities and expected rewards.)

(c) (8 pts) Give the values for states A and B that would be obtained by the batch TD method. Briefly explain how you arrived at your answer. You may express your answer using fractions.
4. (3 pts) **Multiple choice:** In learning methods, a larger step size, $\alpha$, usually means

- a) more rapidly approaching the final performance level
- b) greater error before reaching the final performance level
- c) less residual error at the final performance level
- d) reduced risk of divergence
- e) both a) and d)

5. (3 pts) **Multiple choice:** In TD methods, a larger discount parameter $\gamma$, $0 < \gamma < 1$, means

- a) a closer approximation to the dynamic-programming solution
- b) more concern for immediate rewards relative to later rewards
- c) less concern for immediate rewards relative to later rewards
- d) both a) and b)
- e) both a) and c)

6.9 [6 points] (equations of Double Expected Sarsa) (from SB text)