

DESIGN IMPROVEMENTS IN ASSOCIATIVE MEMORIES FOR CEREBELLAR MODEL ARTICULATION CONTROLLERS (CMAC).

P.C. Edgar An¹, W. Thomas Miller III¹, P.C. Parks²

¹Electrical and Computer Engineering Department, University of New Hampshire, Durham, NH 03824, USA.

²Applied and Computational Mathematics Group, Royal Military College of Science, Shrivenham, Swindon, SN6 8LA, UK.

Abstract

A number of recent improvements to the design of associative memories for CMAC systems are described. These are (i) an improved scheme for allocating C weights to a given input vector in R^N , (ii) design of receptive field shapes within the hypercube associated with an individual weight (including some experimental evaluations of these shapes), (iii) matching the field shapes to the hypercube itself using the concept of "superspheres", (iv) speeding up the convergence of the weight training procedure.

1. INTRODUCTION

The "Cerebellar Model Articulation Controller" (CMAC) was introduced by J.S. Albus [1] in the early 1970s, a concept based in turn on F. Rosenblatt's "perceptron" of the early 1960s [2]. At the heart of the CMAC system is an associative memory which "learns", in a training procedure, the non-linear function $y = f(s_1, s_2, \dots, s_N)$, where $\underline{s} = (s_1, s_2, \dots, s_N)$ is the input or "stimulus" vector in R^N . In the control application $y = y_t$ is usually the scalar output at discrete time t of a non-linear difference equation of a system in which s_1, s_2, \dots, s_N are in fact $y_{t-1}, y_{t-2}, \dots, y_{t-l}, u_{t-1}, u_{t-2}, \dots, u_{t-m}$, where $l+m = N$ and the sequence $\{u_t\}$ is the input to the system. The memory thus "learns the system dynamics".

A key feature of the associative memory is its ability to "generalise", that is to interpolate and extrapolate to some extent from previously learned responses y to given stimulus vectors \underline{s} when new stimulus vectors are encountered during or after completion of the learning process. This is achieved by calculating y as the average sum of a fixed number of C "weights" x_i corresponding to the given input vector \underline{s} . We can represent this choice by ordering the weights in a p-vector as $\underline{x}^T = (x_1, x_2, \dots, x_p)$ and considering a binary "associative vector" \underline{a}^T consisting of 1s in the C positions corresponding to chosen x_i 's and zeros elsewhere. Thus the generalising property is performed by a suitable mapping from the input vector \underline{s} to the association vector \underline{a} . The desired property of this $\underline{s} \rightarrow \underline{a}$ mapping (the "receptive field center placement problem") may be summed up in the following statement (*):

"For input vectors \underline{s}_k lying within a certain neighbourhood of chosen input vector \underline{s}_j the Hamming distance between the corresponding association vectors \underline{a}_k and \underline{a}_j should be linearly proportional to the Euclidean distance between \underline{s}_k and \underline{s}_j ".

2. THE RECEPTIVE CENTER PLACEMENT PROBLEM

The receptive field centers in the standard CMAC procedure developed by Albus [1] are distributed on the grid of each of C hyperplanes with receptive field width of C . Each of two consecutive hyperplanes are relatively offset by one quantization unit in each dimension (fig 1).

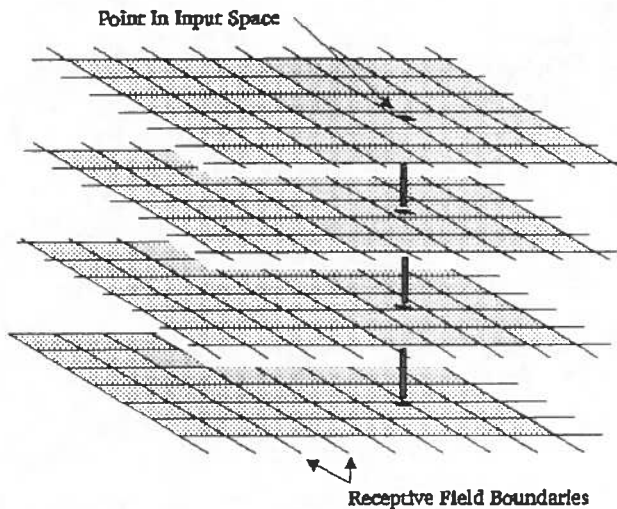


Figure 1 The organization of centres for the two-dimensional CMAC with C of 4. The dark square in each plane indicates the activated center. ($N=2$ $C=4$)

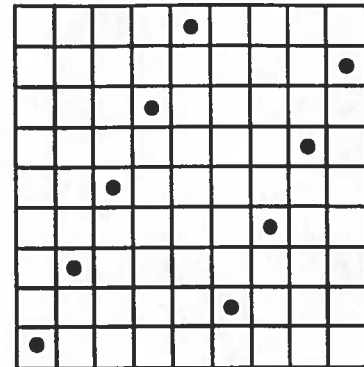


Figure 2 The solid dots show the centers in the two-dimensional hypercube (square in this case). The generalisation level was 9, and D was $\langle 1, 2 \rangle$. ($N=2$, $C=9$)

This is equivalent to distributing the centers along the hyper-diagonal inside a hypercube, a building block of the lattice or "basic association cell". In the standard CMAC, the number of centers inside the hypercube is equal to C . Increasing C only increases the weight density along the hyper-diagonal. Although the distribution appears uniform in a global view, it is highly non-uniform inside the hypercube, so the statement (*) of the introduction is not achieved.

Despite the non-uniform coverage, the hyper-diagonal center distribution has a uniform projection of centers on the axes of the reference hypercube, which is a desirable feature. Also the regularity of the center placement eliminates the exhaustive search for neighbouring centers.

We have developed a rule which provides a more uniform distribution of receptive fields while preserving desirable features of the standard CMAC. To explain the rule, we adopt a modular arithmetic expression to define the center co-ordinates. The co-ordinates of the m th center inside the hypercube is defined as $[m \cdot d_1 \% C, m \cdot d_2 \% C, \dots, m \cdot d_N \% C]$, where $\%$ is the modulo operator, and m ranges from 0 to $C-1$. It is easily seen that the first center is always fixed at the origin $[0, 0, \dots, 0]$ by this co-ordinate definition. To be precise, a displacement vector D is defined as $\langle d_1, d_2, \dots, d_N \rangle$, where d_1 is set to 1. The center distribution is completely described by the vector D . D is simply

a unity vector for the standard CMAC. This rule provides a better offsetting scheme for the hyperplanes which still guarantees a uniform projection of centers on the axes and we introduce the rule R1:

$$\alpha \cdot d_i \neq \gamma \cdot C, \quad \text{where } 0 \leq i < C, \quad 0 < d_i < C/2, \quad d_i, \alpha, \gamma \in I.$$

All d_i 's which satisfy R1 are potential candidates for the components in D. The range of d_i is limited up to $C/2$ due to the mirror image symmetry. If there are more choices for d_i than N, d_i should be chosen in such a way to maximize the deviation among all chosen d_i 's. If there are fewer choices for d_i than N, d_i should be chosen so that the d_i 's selected with roughly equal frequencies. For example, for a CMAC with C of 9 and N of 2, possible d_i are {1,2,4}. A vector D of <1,2> satisfies all the constraints. If this is chosen, the co-ordinates for the centers are [0,0], [1,2], [2,4], and so forth. The arrangement of these centers is shown in fig. 2.

Parks and Militzer [4] generated optimal vectors D for many combinations of C and N by using an exhaustive search. The evaluation was based on maximising the minimum distance between two nearest cell-center neighbours. Reference [4] gives tables of the optimised vectors < d_1, d_2, \dots, d_N > for $2 \leq N \leq 10$ and $2 \leq C \leq 50$.

The issue of the ultimate ideal center distribution is still an open question.

3. RECEPTIVE FIELD SHAPES

Two inter-related undesirable features which occur when using a rectangular receptive field function are that the field strength is insensitive to the input position within the field, and it also exhibits a slow learning phenomenon at particular spatial frequencies [3]. A better receptive field should be tapered from the field center to the boundary so that the network output is continuous rather than piecewise constant. In this paper, a linearly tapered field shape was chosen for study (although other field shapes, like a Gaussian or cosine function, can generate smoother functions). Given that the field shape on every axis should be tapered, there are many ways to form a linearly tapered field shape in a multi-dimensional input space. One possible choice is to have a field shape with a radial basis contour. In an $N=2$ input space, the field shape is a cone.

As the Euclidean distance from the center to the far corner inside the hypercube is $(\sqrt{N}) \cdot (C/2)$, the corresponding field strength becomes very weak in the receptive field corners in a multi-dimensional space. In view of this, an alternative field contour (square basis) is also used. Under this square contour, the overall field strength of each center for any given input is defined as the minimum projected field strength among all the axes. In an $N=2$ input space, the field shape is a pyramid. Although this field shape introduces discontinuity along the hyper-diagonal, the magnitude of the field strength is now independent of the input dimension. In the next section, the results from these two chosen field shapes are compared.

4. EXPERIMENTAL EVALUATION

The uniformity of receptive field coverage can best be evaluated in terms of function approximation. The space-spanning function of interest in this paper is a constant output. One way to evaluate the uniformity of coverage is to set all the weights equal to 1, and to evaluate the network output at many random points in the hypercube of side C in the input space. The network is simply the sum of C field strengths. If the sum is not a constant over the region, this means the network is more responsive to some inputs than others: this is undesirable.

Experiments were done on a CMAC with C from 2 to 50 both with $N=3$ and $N=100$ input spaces. Linearly tapered radial basis and square basis receptive field functions were used with $R1$ center placement. These functions were then compared with the modified standard CMAC (diagonal center placement and the linearly tapered field shape with the square basis contour). The network outputs were evaluated at 1000 random inputs inside the hypercube, and the ratio of the standard deviation to the mean of these responses was computed. The ratio for the standard CMAC with the rectangular field shape produces exactly zero for any input because of a fixed generalization for each input).

The ratios obtained for three different field shapes in a $N=3$ input space is shown in fig 3. The ratios for the modified standard CMAC were fairly independent of C . This is because increasing C makes the coverage more non-uniform. On the other hand, increasing C also increases the field strength mean. The new center ($R1$) placement with the linearly tapered radial basis field shape produced smaller ratios for larger C 's. The $R1$ placement with the linearly tapered square basis field shape produced the smallest ratio. Also, the ratios were very sensitive to C .

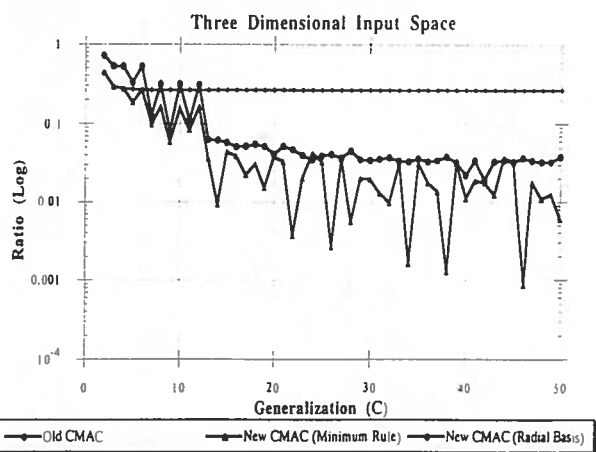


Figure 3 The ratio of the deviation to the mean of the total summed field strengths evaluated at 100 random location in the $N=3$ input space. The horizontal axis shows the generalization level. ($N=3$, C variable)

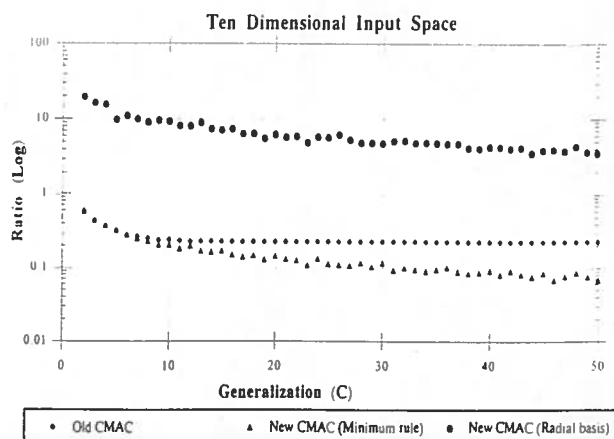


Figure 4 The ratio of the deviation to the mean of the total summed field strengths evaluated at 1000 random locations in the $N=100$ input space. The horizontal axis shows the generalization level. ($N=100$, C variable)

Fig 4 shows the ratios for the same three field shapes in an N=10 input space. The ratios for the modified standard CMAC were still invariant of C for the same reason. The linearly tapered radial basis function, however, produced the biggest ratios. This is because the centers at far corners with respect to the input contributed insignificant (or zero) field strengths, which caused the overall mean strength to be very small. The linearly tapered square basis function became less sensitive to C, but still produced the smallest ratio among all field shapes.

The R1 center placement produces a better fit without introducing any penalty in computation. In addition, the radial basis contour is found to be undesirable, which suggests that the square basis contour is best suited to CMAC due to the hypercubic nature of the building block, unless a more sophisticated field shape such as that suggested in Section 5 is used.

5. USE OF SUPERSPHERES

A problem which arises with the receptive field shapes described in section 3 above is that discontinuities in partial derivative of the stored function f can arise for example when crossing joint lines of the pyramid for the N=2 input space described in Section 3. A natural scheme to match receptive field shapes to the CxCx...xC N dimensional hypercube which forms the boundaries of a receptive association cell is to use a "supersphere" [5]. The center the origin and general equation of a supersphere center the origin and of radius R in N dimensions, with coordinates $\xi_1, \xi_2, \dots, \xi_N$ is given by

$$|\xi_1|^n + |\xi_2|^n + \dots + |\xi_N|^n = R^n \text{ where } n \text{ is a real number with } n \geq 2.$$

The superspherical contours may be combined with a cosine field shape to give a field shape $z(\xi_1, \xi_2, \dots, \xi_N)$ defined by the transcendental equation

$$|\xi_1/\frac{1}{2}C|^{1+1/\sqrt{z}} + \dots + |\xi_N/\frac{1}{2}C|^{1+1/\sqrt{z}} = \{1/\pi \cos^{-1}(2z-1)\}^{1+1/\sqrt{z}}$$

Here $(\xi_1, \xi_2, \dots, \xi_N)$ is a local coordinate system based on an origin at the center of the CXCX...XC N-dimensional hypercube. For a given point (ξ_1, \dots, ξ_N) z may be found by a Newton-type iteration using as a first approximation

$$z = z_0 = \frac{1}{2} + \frac{1}{2} \cos \left(\pi \max_{1 \leq i \leq N} |\xi_i/\frac{1}{2}C| \right)$$

This scheme is probably as far as one would wish to go in devising sophisticated receptive field shapes at the present time. The shape of z for the case N = 2 is sketched in Fig 5. The contours of z in Fig 5 vary from circles in the centre to very nearly squares on the boundaries of the basic C x C association cell.

6. SPEEDING UP CONVERGENCE OF THE LEARNING ALGORITHM

Here it will be simply recorded that Parks and Militzer [6] have done much work on speeding up the learning algorithm - devised

originally by Albus [1] as a "Hebbian learning" process. Their recommended "maximum error" algorithm reduces to zero the largest residual error occurring among a set of linear equations defining the values of the weights x_i . This algorithm is in fact a variation of a basic projection algorithm for solving least mean squares (LMS) problems devised in 1937 by S. Kaczmarz [7].

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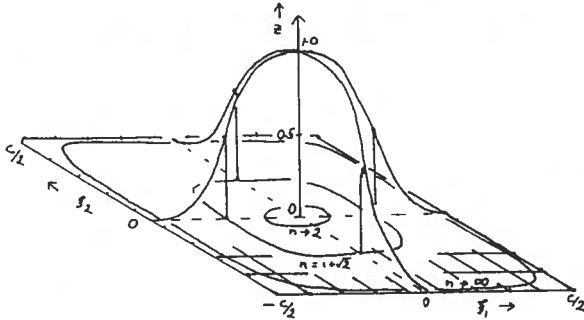


Figure 5 (left). The receptive field shape $z(\xi_1, \xi_2)$ described in Section 5, ($N=2$ $C=9$).

8. REFERENCES

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