$$v_\pi(s) \approx \theta^\top \phi(s) \doteq v_\theta(s) \in \mathbb{R} \qquad v_\theta \in \mathbb{R}^{|\mathcal{S}|}$$

$$d : \mathcal{S} \to \mathbb{R}, \qquad \sum_{i=1}^{n} d(s) = 1$$

$$\forall v : \mathcal{S} \to \mathbb{R}, \qquad \|v\| \doteq \sum_{s\in\mathcal{S}} d(s)v(s)^2$$

$$\Pi v \doteq v_\theta \;\text{ where }\; \theta = \arg\min_\theta \|v - v_\theta\| . \tag{1}$$

For a linear function approximator, the projection operator is linear, which implies that it can be represented as an $|\mathcal{S}| \times |\mathcal{S}|$ matrix:

$$\Pi \doteq \Phi \left(\Phi^\top D \Phi\right)^{-1} \Phi^\top D, \tag{2}$$

where D denotes the $|\mathcal{S}| \times |\mathcal{S}|$ diagonal matrix with d on the diagonal, and Φ denotes the $|\mathcal{S}| \times n$ matrix whose rows are the feature vectors $\phi(s)^\top$, one for each state s:

$$D \doteq \begin{bmatrix} d(1) & & & 0 \\ & d(2) & & \\ & & \ddots & \\ 0 & & & d(|\mathcal{S}|) \end{bmatrix}, \qquad \Phi \doteq \begin{bmatrix} -\phi(1)^\top- \\ -\phi(2)^\top- \\ \vdots \\ -\phi(|\mathcal{S}|)^\top- \end{bmatrix}. \tag{3}$$

(Formally, the inverse in (2) may not exist, in which case the pseudoinverse is substituted.) Using these matrices, the squared norm of a vector can be written

$$\|v\| = v^\top D v, \tag{4}$$

and the approximate linear value function can be written

$$v_\theta = \Phi\theta. \tag{5}$$

$$\text{MSVE}(\theta) \doteq \|v_\theta - v_\pi\| \tag{6}$$

$$\text{MSRE}(\theta) \doteq \mathbb{E}_\pi\Big[\big(v_\theta(S_t) - G_t\big)^2\Big]$$

Bellman operator for policy π:

$$(B_\pi v)(s) \doteq \sum_{a\in\mathcal{A}} \pi(s,a) \left[r(s,a) + \gamma \sum_{s'\in\mathcal{S}} p(s'|s,a)v(s') \right], \qquad \forall s \in \mathcal{S},\ \forall v : \mathcal{S} \to \mathbb{R}, \tag{7}$$

which can also be written,

$$B_\pi v = r_\pi + \gamma P_\pi v, \qquad \forall v : \mathcal{S} \to \mathbb{R}, \tag{8}$$

where $r_\pi \in \mathbb{R}^{|\mathcal{S}|}$ s.t. $[r_\pi]_s = \sum_{a\in\mathcal{A}} \pi(s,a) r(s,a)$,
and $P_\pi \in \mathbb{R}^{|\mathcal{S}|} \times \mathbb{R}^{|\mathcal{S}|}$ s.t. $[P_\pi]_{ji} = \sum_{a\in\mathcal{A}} \pi(i,a)p(j|i,a)$ Bellman equation:

$$v_\pi = B_\pi v_\pi, \tag{9}$$

Bellman error:

$$\bar{\delta}_\theta \doteq B_\pi v_\theta - v_\theta. \tag{10}$$

$$\text{MSBE}(\theta) \doteq \left\|\bar{\delta}_\theta\right\|, \tag{11}$$

$$\text{MSTDE}(\theta) \doteq \mathbb{E}_\pi\Big[\big(R_{t+1} + \gamma v_\theta(S_{t+1}) - v_\theta(S_t)\big)^2\Big]$$